Advanced Statistics

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Problem Set 2. Probability theory - counting, conditional probability, independence, Random variables

References

- FPP: Statistics (4/e), Freedman, Pisani, Purves
- Wasserman: All of Statistics, Springer, 2003
- JWHT: An Introduction to Statistical Learning with Applications in R, James, Witten, Hastie and Tibshirani, Springer, 2017
- VS: An Introduction to R, Venables and Smith

1. FPP page 227: Exercise set B

2. FPP page 230: Exercise set C

3. FPP page 230: Exercise set D

4. FPP page 235: 3, 5, 6, 7, 8, 9, 10

5. FPP pages 240-2411: 1, 2, 3, 4

6. FPP pages 242-243: Exercise set B

7. FPP pages 246-247: Exercise set C

8. FPP pages 250-251: Exercise set D

9. FPP pages 252-254: 1, 3, 4, 5, 7, 8, 10, 12

10. Describe the Paradox of Chevalier de Méré. Resolve the paradox.

11. In how many ways can eight colored beads, all colored differently, be arranged on (i) a straight wire and (ii) a circular necklace?

12. A firm has to choose seven people from its R&D team of eleven to send to a conference on computer systems. How many ways are there of doing this

i. when there are no restrictions?

ii. when two of the team are so indispensable that only one of them can be permitted to go?

iii. when it is essential that a certain member of the team goes?

13. Prove that the number of ways that a group of r objects can be chosen from n objects using sampling with replacement where the order of selection does not matter is given by $\binom{n+r-1}{r}$.

14. State and prove inclusion-exclusion principle.

15. Wasserman, pages 13-16, problems 1, 3, 4, 5, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20

16. In a factory producing compact discs, the total quantity of defective items found in a given week is 14%. It is suspected that the majority of these come from two machines, X and Y. An inspection shows that 8% of the output from X and 4% of the output from Y is defective. Furthermore, 11% of the overall output came from X and 23% from Y. A CD is chosen at random and found to be defective. What is the probability that it came from either X or Y?

17. A firm buys 1000 hard disks from two vendors. It buys 900 disks from V_1 and 100 from V_2. The probabilities of shipping a defective disk are 0.01 and 0.005 for V_1 and V_2, respectively. One disk is taken randomly from the lot and is found working. What is the probability that the second disk taken randomly will also be found working? Explain the result.

18. An office secretary puts n letters, all addressed to different individuals, in n labeled envelopes randomly. What is the probability that at-least one of the letters is in correctly labeled envelope when $n = 4$? What happens when n gets large?

19. Show that if three events A, B and C are independent, then $A \cup B$ is independent of C.

20. *Gambler's ruin.* A gambler needs to raise N Rupees and has k Rupees in hand. He bets 1 Rupee on a fair coin toss where he wins 1 Rupee if head shows up and looses the same amount if tail shows up. What is the probability that he looses all his money?

21. Programming exercise: Wasserman, page 16, problem 21