Advanced Statistics, 2022

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Problem Set 4. Expectation, moments, distributions, moment generating functions

References

- FPP: Statistics (4/e), Freedman, Pisani, Purves
- Wasserman: All of Statistics, Springer, 2003
- JWHT: An Introduction to Statistical Learning with Applications in R, James, Witten, Hastie and Tibshirani, Springer, 2017
- VS: An Introduction to R, Venables and Smith

1. Find expectation and variance of a random variable distributed according to

- Binomial distribution with parameters (n, p)
- Geometric distribution with parameter p
- Poisson distribution with parameter λ
- Uniform density in $[0, 1]$
- Exponential density with parameter λ
- Gaussian density with parameters $(0, 1)$

2. Wasserman, Chap. 3, problem 23

3. Let $X_1 \sim \mathcal{P}(\lambda_1)$ and $X_2 \sim \mathcal{P}(\lambda_2)$ be independent Poisson random variables. Show that $X_1 + X_2 \sim \mathcal{P}(\lambda_1 + \lambda_2)$.

4. Let X and Y be independent random variables. Show that $\text{Var}(X - Y) = \text{Var}(X + Y)$.

5. A gambler offers you to bet on roll of a fair die where you bet on a number of your choice. For every 100 Rupees of your stake, the gambler will give you 400 if you win the bet. Should you take the offer? Why or why not?

6. Wasserman, Chap. 3, problem 3

7. Wasserman, Chap. 2, All examples of sections 2.5 and 2.6

8. Suppose that the joint probability density function of random variables X and Y is

$$f(x, y) = \begin{cases} c(x^2 + 2y), & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Determine c and find the marginal density functions. Are X and Y independent? Explain your answer.

9. A box contains 10 balls, out of which three are red, three are green and four are blue. We randomly select 5 balls with replacement. What is the probability of selecting 2 red and 3 blue balls?

10. The waiting time T at a bus stop is often modeled with p.d.f. of the form $f(t) = \lambda e^{-\lambda t}$. Show that the waiting time is memoryless, i.e., $Pr(T > t + k | T > t) = Pr(T > k)$. Suppose that the average waiting time is 10 minutes. What is λ?