

# CSE 518 - Artificial Intelligence

## Homework

Instructor: Shashi Prabh

### Chapter 3. Solving Problems by Searching

3.1 Explain why problem formulation must follow goal formulation.

3.2 Give a complete problem formulation for each of the following problems. Choose a formulation that is precise enough to be implemented.

- a. There are six glass boxes in a row, each with a lock. Each of the first five boxes holds a key unlocking the next box in line; the last box holds a banana. You have the key to the first box, and you want the banana.
- b. You start with the sequence ABABAECCEC, or in general any sequence made from A, B, C, and E. You can transform this sequence using the following equalities:  $AC = E$ ,  $AB = BC$ ,  $BB = E$ , and  $Ex = x$  for any  $x$ . For example, ABBC can be transformed into AEC, and then AC, and then E. Your goal is to produce the sequence E.
- c. There is an  $n \times n$  grid of squares, each square initially being either unpainted floor or a bottomless pit. You start standing on an unpainted floor square, and can either paint the square under you or move onto an adjacent unpainted floor square. You want the whole floor painted.
- d. A container ship is in port, loaded high with containers. There 13 rows of containers, each 13 containers wide and 5 containers tall. You control a crane that can move to any location above the ship, pick up the container under it, and move it onto the dock. You want the ship unloaded.

3.3 You have a  $9 \times 9$  grid of squares, each of which can be colored red or blue. The grid is initially colored all blue, but you can change the color of any square any number of times. Imagining the grid divided into nine  $3 \times 3$  sub-squares, you want each sub-square to be all one color but neighboring sub-squares to be different colors.

- a. Formulate this problem in the straightforward way. Compute the size of the state space.
- b. You need color a square only once. Reformulate, and compute the size of the state space. Would breadth-first graph search perform faster on this problem than on the one in (a)? How about iterative deepening tree search?
- c. Given the goal, we need consider only colorings where each sub-square is uniformly colored. Reformulate the problem and compute the size of the state space.
- d. How many solutions does this problem have?

- e. Parts (b) and (c) successively abstracted the original problem (a). Can you give a translation from solutions in problem (c) into solutions in problem (b), and from solutions in problem (b) into solutions for problem (a)?

**3.4** Consider the  $n$ -queens problem using the “efficient” incremental formulation given in Section 4.1. Explain why the state space has at least  $\sqrt[3]{n!}$  states and estimate the largest  $n$  for which exhaustive exploration is feasible. (*Hint*: Derive a lower bound on the branching factor by considering the maximum number of squares that a queen can attack in any column.)

**3.5** The missionaries and cannibals problem is usually stated as follows. Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people. Find a way to get everyone to the other side without ever leaving a group of missionaries in one place outnumbered by the cannibals in that place. This problem is famous in AI because it was the subject of the first paper that approached problem formulation from an analytical viewpoint.

- a. Formulate the problem precisely, making only those distinctions necessary to ensure a valid solution. Draw a diagram of the complete state space.
- b. Implement and solve the problem optimally using an appropriate search algorithm. Is it a good idea to check for repeated states?
- c. Why do you think people have a hard time solving this puzzle, given that the state space is so simple?

**3.6** Define in your own words the following terms: state, state space, search tree, search node, goal, action, transition model, and branching factor.

**3.7** What’s the difference between a world state, a state description, and a search node? Why is this distinction useful?

**3.8** An action such as *Go(Sibiu)* really consists of a long sequence of finer-grained actions: turn on the car, release the brake, accelerate forward, etc. Having composite actions of this kind reduces the number of steps in a solution sequence, thereby reducing the search time. Suppose we take this to the logical extreme, by making super-composite actions out of every possible sequence of *Go* actions. Then every problem instance is solved by a single super-composite action, such as *Go(Sibiu)Go(Rimnicu Vilcea)Go(Pitesti)Go(Bucharest)*. Explain how search would work in this formulation. Is this a practical approach for speeding up problem solving?

**3.9** Does a finite state space always lead to a finite search tree? How about a finite state space that is a tree? Can you be more precise about what types of state spaces always lead to finite search trees?

**3.10** Consider the vacuum-world problem defined in Fig. 3.2.

- a. Which of the algorithms defined in this chapter would be appropriate for this problem? Should the algorithm use tree search or graph search?
- b. Apply your chosen algorithm to compute an optimal sequence of actions for a  $3 \times 3$  world whose initial state has dirt in the three top squares and the agent in the center.
- c. Construct a search agent for the vacuum world, and evaluate its performance in a set of  $3 \times 3$  worlds with probability 0.2 of dirt in each square. Include the search cost as well as path cost in the performance measure, using a reasonable exchange rate.
- d. Compare your best search agent with a simple randomized reflex agent that sucks if there is dirt and otherwise moves randomly.

e. Consider what would happen if the world were enlarged to  $n \times n$ . How does the performance of the search agent and of the reflex agent vary with  $n$ ?

**3.11** Compare the performance of  $A^*$  and RBFS on a set of randomly generated problems in the 8-puzzle (with Manhattan distance) and TSP domains. Discuss your results. What happens to the performance of RBFS when a small random number is added to the heuristic values in the 8-puzzle domain?



**3.12** Sometimes there is no good evaluation function for a problem but there is a good comparison method: a way to tell whether one node is better than another without assigning numerical values to either. Show that this is enough to do a best-first search. Is there an analog of  $A^*$  for this setting?