CSE 518 - Artificial Intelligence
Homework

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Chapter 16. Making Simple Decisions

16.1 Chris considers four used cars before buying the one with maximum expected utility. Pat considers ten cars and does the same. All other things being equal, which one is more likely to have the better car? Which is more likely to be disappointed with their car's quality? By how much (in terms of standard deviations of expected quality)?

16.2 In 1713, Nicolas Bernoulli stated a puzzle, now called the St. Petersburg paradox which works as follows. You have the opportunity to play a game in which a fair coin is tossed repeatedly until it comes up heads. If the first heads appears on the nth toss, you win 2^n dollars.

a. Show that the expected monetary value of this game is infinite.

b. How much would you, personally, pay to play the game?

c. Nicolas's cousin Daniel Bernoulli resolved the apparent paradox in 1738 by suggesting that the utility of money is measured on a logarithmic scale (i.e., $U(S_n) = a \log_2 n + b$, where S_n is the state of having \$$n$). What is the expected utility of the game under this assumption?

d. What is the maximum amount that it would be rational to pay to play the game, assuming that one's initial wealth is \$$k$?

16.3 The Surprise Candy Company makes candy in two flavors: 75% are strawberry flavor and 25% are anchovy flavor. Each new piece of candy starts out with a round shape; as it moves along the production line, a machine randomly selects a certain percentage to be trimmed into a square; then, each piece is wrapped in a wrapper whose color is chosen randomly to be red or brown. 70% of the strawberry candies are round and 70% have a red wrapper, while 90% of the anchovy candies are square and 90% have a brown wrapper. All candies are sold individually in sealed, identical, black boxes.

Now you, the customer, have just bought a Surprise candy at the store but have not yet opened the box. Consider the three Bayes nets in Figure 1.

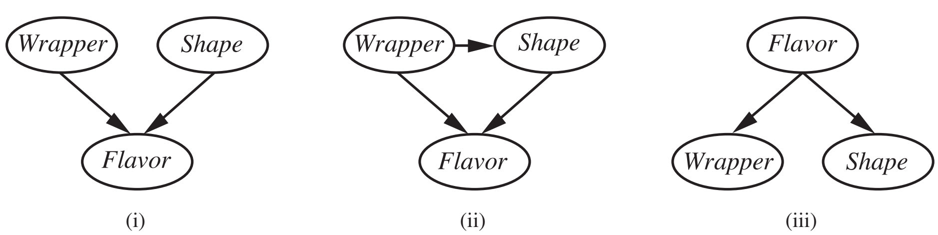

Figure 1: Three proposed Bayes nets for the Surprise Candy problem, Exercise 16.3.

a. Which network(s) can correctly represent $\mathbf{P}(Flavor, Wrapper, Shape)$?

b. Which network is the best representation for this problem?

c. Does network (i) assert that $\mathbf{P}(Wrapper|Shape) = \mathbf{P}(Wrapper)$?

d. What is the probability that your candy has a red wrapper?

e. In the box is a round candy with a red wrapper. What is the probability that its flavor is strawberry?

f. A unwrapped strawberry candy is worth s on the open market and an unwrapped anchovy candy is worth a. Write an expression for the value of an unopened candy box.

g. A new law prohibits trading of unwrapped candies, but it is still legal to trade wrapped candies (out of the box). Is an unopened candy box now worth more than less than, or the same as before?

16.4 Tickets to a lottery cost $1. There are two possible prizes: a $10 payoff with probability 1/50, and a $1,000,000 payoff with probability 1/2,000,000. What is the expected monetary value of a lottery ticket? When (if ever) is it rational to buy a ticket? Be precise—show an equation involving utilities. You may assume current wealth of $$k$ and that $U(S_k) = 0$. You may also assume that $U(S_{k+10}) = 10 \times U(S_{k+1})$, but you may not make any assumptions about $U(S_{k+1,000,000})$. Sociological studies show that people with lower income buy a disproportionate number of lottery tickets. Do you think this is because they are worse decision makers or because they have a different utility function? Consider the value of contemplating the possibility of winning the lottery versus the value of contemplating becoming an action hero while watching an adventure movie.

16.5 Assess your own utility for different incremental amounts of money by running a series of preference tests between some definite amount M_1 and a lottery $[p, M_2; (1-p), 0]$. Choose different values of M_1 and M_2, and vary p until you are indifferent between the two choices. Plot the resulting utility function.

16.6 How much is a micromort worth to you? Devise a protocol to determine this. Ask questions based both on paying to avoid risk and being paid to accept risk.

16.7 Consider a student who has the choice to buy or not buy a textbook for a course. We'll model this as a decision problem with one Boolean decision node, B, indicating whether the agent chooses to buy the book, and two Boolean chance nodes, M, indicating whether the student has mastered the material in the book, and P, indicating whether the student passes the course. Of course, there is also a utility node, U. A certain student, Sam, has an additive utility function: 0 for not buying the book and -$100 for buying it; and $2000 for passing the course and 0 for not passing. Sam's conditional probability estimates are as follows:

$$
\begin{array}{ll}
P(p|b,m) = 0.9 & P(m|b) = 0.9 \\
P(p|b,\neg m) = 0.5 & P(m|\neg b) = 0.7 \\
P(p|\neg b,m) = 0.8 & \\
P(p|\neg b,\neg m) = 0.3 &
\end{array}
$$

You might think that P would be independent of B given M, But this course has an open-book final—so having the book helps.

a. Draw the decision network for this problem.

b. Compute the expected utility of buying the book and of not buying it.

c. What should Sam do?

16.8 A used-car buyer can decide to carry out various tests with various costs (e.g., kick the tires, take the car to a qualified mechanic) and then, depending on the outcome of the tests, decide which car to buy. We will assume that the buyer is deciding whether to buy car c_1, that there is time to carry out at most one test, and that t_1 is the test of c_1 and costs $50.

A car can be in good shape (quality q^+) or bad shape (quality q^-), and the tests might help indicate what shape the car is in. Car c_1 costs $1,500, and its market value is $2,000 if it is in good shape; if not, $700 in repairs will be needed to make it in good shape. The buyer's estimate is that c_1 has a 70% chance of being in good shape.

a. Draw the decision network that represents this problem.

b. Calculate the expected net gain from buying c_1, given no test.

c. Tests can be described by the probability that the car will pass or fail the test given that the car is in good or bad shape. We have the following information:
$P(pass(c_1, t_1)|q^+(c_1)) = 0.8$
$P(pass(c_1, t_1)|q^-(c_1)) = 0.35$
Use Bayes' theorem to calculate the probability that the car will pass (or fail) its test and hence the probability that it is in good (or bad) shape given each possible test outcome.

d. Calculate the optimal decisions given either a pass or a fail, and their expected utilities.

e. Calculate the value of information of the test, and derive an optimal conditional plan for the buyer.